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ORTHOGONALITY AND NORMALIZATION OF TORSIONAL MODES OF VIBRATION OF SOLID ELASTIC SPHERES

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In this paper the work of Lamb on the free torsional vibrations of spheres (or "Vibrations of the First Class") has been extended. In particular the conditions for the orthogonality of the normal modes have been established. These conditions have allowed the normalized displacements to be presented as a function of the radial distance from the centre of the sphere. Three-dimensional diagrams have been devised which show the displacements of the surface of the sphere. These modal diagrams also show the nodes and nodal lines.

1. INTRODUCTION

The vibrations of spheres have been studied for over one hundred years. A dynamical theory of the motion of an elastic sphere has been developed which can be applied to both large and small spheres. The theory is of value in explaining the oscillations of the earth [1–3] and can also be used with very small spheres in the determination of elastic constants [4]. The present paper is part of a larger study of the sound and vibration from spheres subjected to impulsive loading [5].

The main guidance for the study of the vibrations of a solid, elastic sphere was provided by Lamb [6]. Lamb formed the equations of motion for small vibrations of a sphere in terms of rectangular co-ordinates. Chree [7] considered the same problem in spherical co-ordinates. Solution of the equations of motion led to two frequency equations and two types of vibration. In one type of vibration there is no change in volume of the sphere, i.e., the dilatation is zero, and the radial displacements are zero. Lamb referred to this type as "Vibrations of the First Class". In the present paper they are called torsional vibrations. Distortion of the spheres occurs in "Vibrations of the Second Class", or spheroidal vibrations. This article is concerned only with torsional vibrations. A subsequent paper will consider spheroidal vibrations.

If the equations of motions are transformed into spherical polar co-ordinates (r, θ, ψ), as shown in Figure 1, the solutions for the displacement may be written in terms of

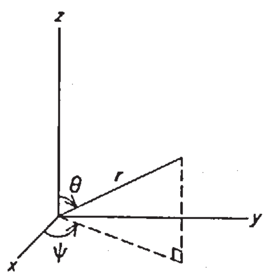

Figure 1. Spherical polar co-ordinates (r, θ, ψ).

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